

Early quantum problem set

- A) In the hydrogen atom, an electron in the energy level $n = 3$ absorbs 1.815×10^{-19} J. What energy level does the electron reach?

$$\Delta E = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Where $\Delta E = 1.815 \times 10^{-19}$ J

$R_H = -2.178 \times 10^{-18}$ J

$n_i = 3$

Solving for n_f : 1.815×10^{-19} J = -2.178×10^{-18} J $\left(\frac{1}{n_f^2} - \frac{1}{3^2} \right)$

$$-0.08333 = \frac{1}{n_f^2} - \frac{1}{9}$$

$$\frac{1}{n_f^2} = 0.02778$$

$$n_f^2 = 36.00$$

$$n_f = 6$$

- B) If this energy corresponds to a photon, what would its wavelength be? Its frequency?

Finding the wavelength through $\frac{1}{\lambda} = R_\infty \left(\frac{1}{n_{in}^2} - \frac{1}{n_{out}^2} \right)$

Or finding the frequency first through $E = h \nu$,
and then λ through $c = \lambda \nu$

$\lambda = 1095$ nm

$\nu = 2.739 \times 10^{14}$ s⁻¹

- C) What part of the electromagnetic spectrum does this photon fall in? Can you name it?

The photon is an infrared emission.

- D) Line spectra were first analyzed with hydrogen. However, scientists soon realized that Bohr's model wasn't directly transposable to other elements. To calculate the changes in energy of non-hydrogen atoms containing one electron (He^+ , Li^{2+} , Be^{3+} , etc), scientists developed this equation:

$$\Delta E = -B \cdot Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

where B is the constant for each particular element

Z is the atomic number of that particular element.

How much energy is involved in a similar transition as in A) in an atom of Li^{2+} for one electron ($B = 27448$ kJ/mol)?

$$\Delta E = -B \cdot Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Where $B = 27448 \text{ kJ/mol}$

$$Z = 3$$

$$n_f = 6$$

$$n_i = 3$$

First, the value of B has to be established in J/atom:

$$\frac{27448 \text{ kJ}}{1 \text{ mol}} \times \frac{1000 \text{ J}}{1 \text{ kJ}} \times \frac{1 \text{ mol}}{6.022 \times 10^{23} \text{ atoms}} = 4.558 \times 10^{-17} \text{ J}$$

Then, solving

$$\Delta E = -B \cdot Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$\Delta E = 3.418 \times 10^{-17} \text{ J}$$

- E) How does this amount of energy compare to the similar transition in the hydrogen atom? Explain.

The energy required to make a transition from $n = 3$ to $n = 6$ in a Li atom containing one electron is greater as the pull from the nucleus is greater, compared to a hydrogen atom where $Z = 1$.

- F) Given that $4.840 \times 10^{-19} \text{ J}$ are released from a hydrogen atom, on what energy level did the electron start its journey, if it landed in $n = 2$?

$$\Delta E = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Where $\Delta E = -4.840 \times 10^{-19} \text{ J}$

$$R_H = -2.178 \times 10^{-18} \text{ J}$$

$$n_f = 2$$

Solving for n_i : $-4.840 \times 10^{-19} \text{ J} = -2.178 \times 10^{-18} \text{ J} \left(\frac{1}{2^2} - \frac{1}{n_i^2} \right)$

$$0.2222 = \frac{1}{4} - \frac{1}{n_i^2}$$

$$-\frac{1}{n_i^2} = -0.0278$$

$$n_i^2 = 36.00$$

$$n_i = 6$$