Summary — Chapter 1 What is a vector ?

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1. Coordinate Spaces: $\mathbb{R}$, $\mathbb{R}^2$, $\mathbb{R}^3$, $\mathbb{R}^n$

Definition

$\mathbb{R}^n$ = all ordered n-tuples $(x_1, \ldots, x_n)$, $x_i \in \mathbb{R}$. Built from an origin, perpendicular axes, and a unit length.

Space	Notation	# coords	dimension
$\mathbb{R}$	x	1	1
$\mathbb{R}^2$	(x, y)	2	2
$\mathbb{R}^3$	(x, y, z)	3	3
$\mathbb{R}^n$	$(x_1, \ldots, x_n)$	n	n

2. Two Roles for a Vector

Position vector

Marks a **location**: drawn from O, its components are the coordinates of the point.

Velocity vector

Describes a **rate or change**: drawn at the object; only direction and length matter.

Example

Car at $(3, 2, 1)$ in $\mathbb{R}^3$.
$\mathrm{r} = (3, 2, 1)$: arrow from O to that point.

Example

Car moves with $\mathrm{v} = (1, 0, 2)$.
Drawn at the car, tangent to its path — *not* at O.

Same algebra — different meaning

Both r and v are triples combined by the same rules. The *context* determines the role, not the mathematics.

3. Scalar Multiplication

Definition

For $c \in \mathbb{R}$ and $\mathrm{v} = (v_1, \ldots, v_n)$: $c\,\mathrm{v} = (cv_1, cv_2, \ldots, cv_n)$. Geometrically: $c > 1$ stretches, $0 < c < 1$ shrinks, $c < 0$ reverses.

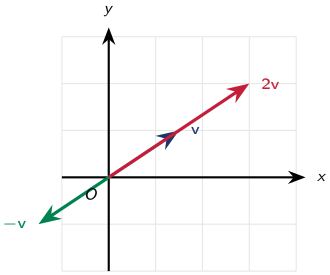
Example

Let $\mathrm{v} = (2, -1, 3) \in \mathbb{R}^3$:

$$3\mathrm{v} = (6, -3, 9), \quad -\mathrm{v} = (-2, 1, -3), \quad 0\mathrm{v} = 0.$$

Let $\mathrm{u} = (1, -2, 0, 4) \in \mathbb{R}^4$:

$$\tfrac{1}{2}\mathrm{u} = \left(\tfrac{1}{2}, -1, 0, 2\right), \quad -2\mathrm{u} = (-2, 4, 0, -8).$$



4. Addition and Subtraction

Definition

$\mathbf{u}+\mathbf{v}=(u_1+v_1,\dots,u_n+v_n), \quad \mathbf{u}-\mathbf{v}=(u_1-v_1,\dots,u_n-v_n).$

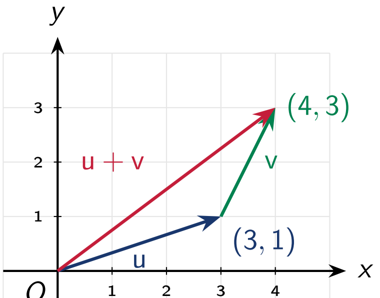
Tip-to-tail: place $\mathbf{v}$ at the tip of $\mathbf{u}$; the sum goes from O to the final tip.
Subtraction: $\mathbf{u}-\mathbf{v}$ points from the tip of $\mathbf{v}$ to the tip of $\mathbf{u}$.

Example

$\mathbf{u}=(3,-1,2)$, $\mathbf{v}=(-1,4,1)$ in $\mathbb{R}^3$:

$$\mathbf{u}+\mathbf{v}=(2,3,3), \qquad \mathbf{u}-\mathbf{v}=(4,-5,1).$$

In $\mathbb{R}^2$: commutativity check:
$(3,1)+(1,2)=(4,3)=(1,2)+(3,1)$ ✓



5. The Vector $\overrightarrow{AB}$ and the Chasles Relation

Vector $\overrightarrow{AB}$

$\overrightarrow{AB}$ is the vector that, **applied at A, takes you to B**:

$$\overrightarrow{AB} = B - A = (b_1 - a_1, \ldots, b_n - a_n).$$

It is *not* tied to A as a base point.

Chasles relation

For any points A, B, C: $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$.

Consequences: $\overrightarrow{AA} = 0$, $\overrightarrow{BA} = -\overrightarrow{AB}$, $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$.

$\overrightarrow{AB}$ in $\mathbb{R}^3$

$A = (1, 0, 2)$, $B = (4, 3, 2)$:

$$\overrightarrow{AB} = (3, 3, 0).$$

Chasles in $\mathbb{R}^2$

$A = (0, 0)$, $B = (3, 1)$, $C = (4, 3)$:

$$\overrightarrow{AB} + \overrightarrow{BC} = (3, 1) + (1, 2) = (4, 3) = \overrightarrow{AC} \checkmark$$

Also: $\overrightarrow{OB} - \overrightarrow{OA} = B - A = \overrightarrow{AB}$.

6. Equal and Collinear Vectors

Equal vectors

u $=$ v iff they are **parallel**, same **length**, same **orientation**. Equivalently: $u_i = v_i$ for all i.

Collinear vectors

u and v are **collinear** iff v $= \lambda$ u for some $\lambda \in \mathbb{R}$. They are parallel but *not* necessarily equal.

- $\lambda > 0$: same orientation.
- $\lambda < 0$: opposite orientation.
- $\lambda = 1$: equal vectors.

Example

Let u $= (2, 1)$ in $\mathbb{R}^2$:

- v $= (2, 1)$: **equal** to u.
- w $= (4, 2) = 2$u: collinear, same orientation, *not* equal.
- p $= (-2, -1) = -$u: collinear, **opposite** orientation.
- q $= (1, 3)$: *not* collinear (no λ works).

7. Equal Vectors on a Parallelogram

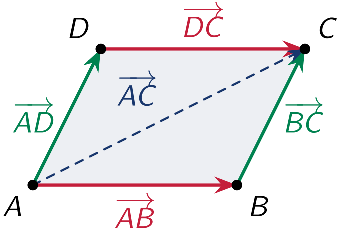
Parallelogram *ABCD*

Opposite sides parallel and equal:

$$\overrightarrow{AB} = \overrightarrow{DC}, \qquad \overrightarrow{AD} = \overrightarrow{BC}.$$

Diagonal = sum of two adjacent sides:

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AD}.$$



Example

$A=(0,0)$, $B=(3,0)$, $C=(4,2)$, $D=(1,2)$:

$\overrightarrow{AB} = (3,0) = \overrightarrow{DC}$ ✓ $\quad \overrightarrow{AD} = (1,2) = \overrightarrow{BC}$ ✓ $\quad \overrightarrow{AC} = (4,2) = \overrightarrow{AB}+\overrightarrow{AD}$ ✓

8. Equal Vectors on a Parallelepiped

Parallelepiped

Built from edges $\mathrm{u}, \mathrm{v}, \mathrm{w}$ from vertex A:

- 12 edges in **3 groups of 4 equal vectors**.
- Main diagonal $A \to G$: $\overrightarrow{AG} = \mathrm{u} + \mathrm{v} + \mathrm{w}$.

Example

$\mathrm{u}{=}(2,0,0)$, $\mathrm{v}{=}(0,3,0)$, $\mathrm{w}{=}(0,0,1)$. $A{=}(0,0,0)$, $G{=}(2,3,1)$.
$\overrightarrow{AG} = (2,0,0) + (0,3,0) + (0,0,1) = (2,3,1)$
All 4 edges parallel to u equal $(2,0,0)$.

Key idea